Analysis of the harpsichord plectrum-string interaction

D. Chadefaux, J-L. Le Carrou
LAM / d'Alembert,
UMR CNRS 7190,
UPMC Univ Paris 06

S. Le Conte
Laboratoire de Recherche
et Restauration
du Musée de la Musique

M. Castellengo
LAM / d'Alembert,
UMR CNRS 7190,
UPMC Univ Paris 06

ABSTRACT

This paper describes a study of string plucking for the harpsichord. Its aim is to provide an experimentally-based analysis of the plectrum-string interaction and to propose some refinements of an existing model. An experimental setup has been designed using a high-speed camera combined with a laser doppler vibrometer and classical audio recordings. This provides accurate estimations of jack and plectrum motion throughout the harpsichord plucking in a realistic musical context. The set of descriptors extracted from these measurements provides typical orders of magnitude of plucking parameters required to feed and validate the investigated model. Results highlight estimations of the instrumentalist's control parameters as well as of the intrinsic plucking parameters according to the performed sequence *tempo*. Besides, a model of the plectrum-string interaction, which takes into account the section variation at the plectrum tip, gives results close to the experimental ones. This model will be of great interest to enquire about harpsichord plectrum voicing.

1. INTRODUCTION

Among all elements of a harpsichord, the voicing process, which directly deals with the excitatory mechanism tuning, has to be investigated. The harpsichord plucking action mostly consists in a piece of wood moving vertically, called the jack, while the instrumentalist's finger is depressing the key. It conveys the plectrum attached to the jack to pluck the string. Although traditionally made of quill [1], the plectrum is nowadays usually made of delrin. During the voicing process, the maker tunes the plectra to obtain the best equilibrium among the different choirs (8 and 4ft), the bass and the trebles. Practically, it implies to carve each plectrum to adjust the plectrum-string interaction according to the instrument's sound and the instrumentalist's touch. Several kinds of voicing can be outlined in the Musée de la musique's collection; the three main ones are: hard, soft and medium.

The physics of harpsichord has already been investigated in several studies, focusing on its elements [1, 2] as well as on its modal behavior [3, 4] or on its dynamics [5]. Further, the question of the plucking action itself has mostly been addressed through modeling [6–8]. Besides, the influence of plectrum geometrical parameters on the produced sound has been theoretically [9] and experimentally [10] analyzed for the harpsichord and the classical guitar, respectively. Although few studies have enhanced the harpsichord plucking modeling by measurements [11, 12], no investigation in a musical context has been carried out as for instance for the concert harp [13, 14].

The present paper describes preliminary results from an ongoing exploratory study of the plectrum voicing. We investigate harpsichord plucking action, within the plectrum-string interaction analysis and modeling. For this purpose, we first present the experiment carried out in order to measure plectrum and string motions during playing. Then, we provide a experimentally-based description of the plucking action. Finally, a plectrum-string interaction model is confronted to experimental data.

2. EXPERIMENTAL PROCEDURE

2.1 Experimental protocole

A set of measurements is fulfilled in order to investigate the harpsichord plucking action. The experimental procedure illustrated in Fig. 1 consists in capturing the plectrum-string interaction during a harpsichord performance. As observations indicate that no motion occurs along the string axis (denoted $\vec{e}_z$ in Fig. 2), the analysis is performed in the ($\vec{e}_x$, $\vec{e}_y$) plane. For this purpose, plectrum motion is measured through a high-speed camera set at 10000 frames per second focusing on the plectrum. A laser doppler vibrometer is also focusing on the associated key to measure its velocity within the performance. The laser beam is focused at about 3 cm from the tip of the key, avoiding obstruction from the musician hand while depressing the key. Simultaneously, acoustical signals are recorded with a microphone, allowing the synchronization of the database.

A skilled harpsichord player has been asked to perform the sequence presented in Fig. 3 with two various *tempi* which have been estimated at about 230 bpm and 440 bpm. Besides, as the high-speed camera sampling rate induces a restricted resolution of 208px×200px representing a 12.3mm×11.8mm area, we were able to only investigate one note. Based on the harpsichord player advices, G4, which fundamental frequency is 392 Hz, is chosen. This process has been carried out with a plectrum in delrin shown in Fig. 4.

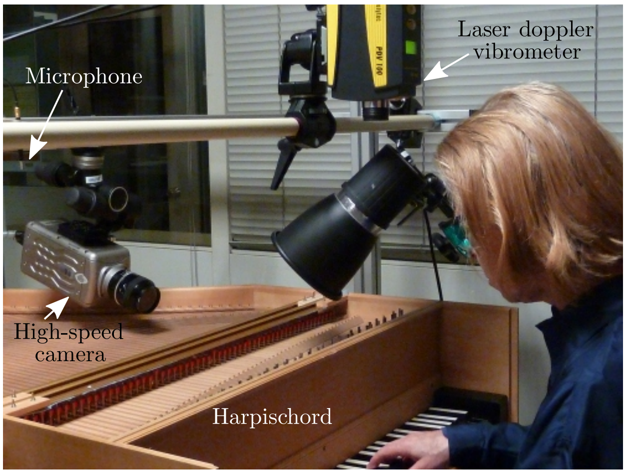

Figure 1. Experimental setup.

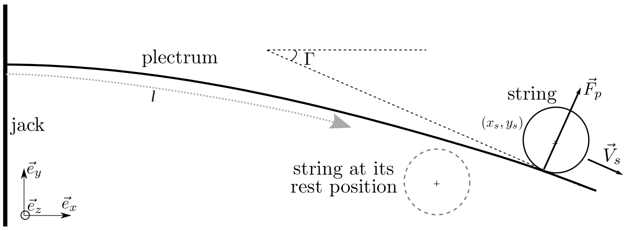

Figure 2. Scheme of the plectrum-string interaction and definition of the parameters.

2.2 Data processing

Fig. 5 shows an example of image obtained through the high-speed camera. The entire sequence is processed to determine a set of plectrum-string interaction descriptors as follow. The desktop background defined at the plectrum rest position is first subtracted from all images. Then, the area of interest containing the plectrum is selected by the user. This template is recursively searched in the entire sequence through a block-matching algorithm model [15]. The string cannot be distinguished in the image because of its contrast. The latter is then refined to obtain a black and white image where only the plectrum appears in white. Hence, based on the framing projection relatively to the measurement plane ($\vec{e}_x$, $\vec{e}_y$, see Fig. 2), the trajectory of the string's plucking point (x_s, y_s) in the latter plane can be extracted from the sequence through its shadow on the plectrum.

Further, the synchronization of acoustical and key velocity signals is needed to point out the instrumentalist control. For this purpose, the onset of each sound event of the score presented in Fig. 3 is highlighted in the acoustical signal, through a standard onset detection algorithm [16, 17]. These instants indicate the strings' release by the plectrum.

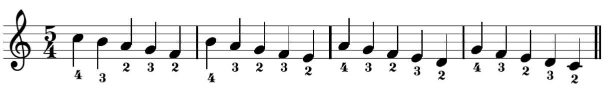

Figure 3. Score of the performed sequence.

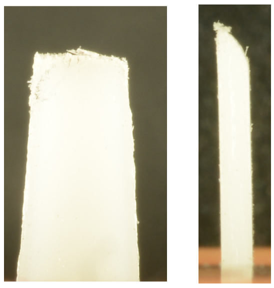

Figure 4. Investigated plectrum: top and side views.

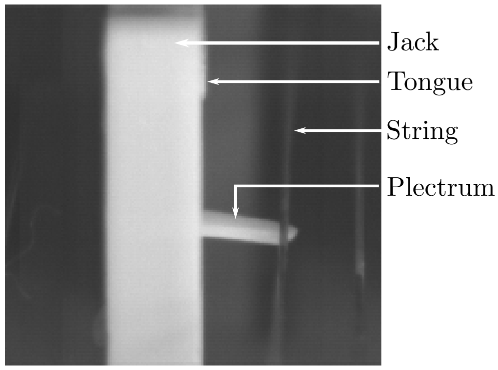

Figure 5. Example of image obtained through the high-speed camera.

3. HARPSICHORD PLUCKING DESCRIPTION

3.1 Temporal phases

The plectrum is directly governed by the instrumentalist to interact with the string through the harpsichord mechanism. Thus, its motion reveals a part of the musician's control while playing as well as determines the initial condition of the string vibration. The laser doppler vibrometer focused on the pressed key directly conveys the velocity pattern of the plectrum at its connection point with the tongue. This investigation combined with the observation of the plectrum motion through the high-speed camera lead to a description into four temporal phases. Based on a harp plucking analysis [13], it can be decomposed as follow:

- The preparatory movement: the plectrum raises from its rest position and approaches the string, $\forall t \in [t_i, t_c]$. Usually, the player first pushes the key until reaching the string to feel its contact with the plectrum. The plectrum is then kept just below the string.
- The sticking phase: the plectrum moves the string in the vertical direction from its rest position, $\forall t \in [t_c, t_s]$;
- The slipping phase: the string slips on the plectrum surface towards its free end, $\forall t \in [t_s, t_r]$;

- The free oscillations of the string, $\forall t > t_r$.

The determination of each instant t_c, t_s and t_r is based on high-speed camera films. The sticking instant t_c is automatically detected at the first instant where the string is moved up by the plectrum. Then, the beginning of the slipping phase t_s is defined when the string's displacement gets a horizontal component. Finally, the release instant is obvious to determine since it corresponds to the beginning of the plectrum vibrations. Note that t_r is manually estimated. Although the detection of t_c and t_r are straightforward, t_s is more complicated to highlight.

In the present case, we estimate the entire duration of the harpsichord plucking action at 12 ± 2 ms and at 7.9 ± 0.6 ms for the normal and the fast *tempi*, respectively. The measurements reported in the literature are about 80 ms [11] and 25 ms [12]. Differences between these orders of magnitude can be explained by the various musical contexts considered. Further, the slipping phase duration has not been pointed out in these previous studies. However, based on the plucking analysis, we measure that the slipping phase lasts about 36 ± 3 % and 26 ± 4 % of the entire plectrum-string interaction duration for the normal and the fast *tempi*, respectively. The small uncertainties, computed over each played note with a 95 % confidence interval, tend to indicate that the measurement protocol is reliable. Eventually, remark that the first note has not been taken into account since it corresponds to the instrumentalist adaptation to the plectrum, which implies a longer interaction: 26 ms, i.e. about two times longer than the following plucking actions.

3.2 Jack velocity

The jack motion is directly measured through the high-speed camera during the plectrum-string interaction. The linear regression associated to its vertical displacement indicates that the jack velocity is constant during plucking. Indeed, the computed coefficients of determination are 0.97 ± 0.02, and 0.99 ± 0.00 for the normal and the fast *tempi*, respectively. Eventually, the jack velocities are estimated in the two former contexts at 0.10 ± 0.05m/s and 0.25 ± 0.03m/s. These estimations are relevant regarding the *tempo* of the played sequence: the higher the *tempo*, the higher the jack velocity. Besides, it confirms that the instrumentalist control is repeatable along a musical sequence, as expected for an expert gesture [18]. Finally, these measurements are consistent with those previously measured or used to fed a harpsichord plucking model [8, 11]. However, note that these values are highly dependent on the plectrum's shape and the material as well as on the string's properties.

Because of the arduousness of the high-speed camera films post-processing, the laser doppler vibrometer is used to get an insight on the instrumentalist's control during the entire sequence. Fig. 6 shows the G-key velocity curve measured on one plucking action in a musical context. This curve has been chosen because of its representativeness among the whole database. It indicates that the player depresses first the key until the plectrum reaches the string $\forall t \in [t_i; t_c]$.

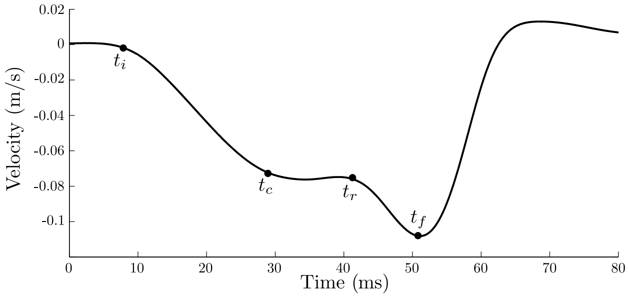

Figure 6. G-key velocity curve during a plucking action and its preparation. t_i corresponds to the instant where the finger starts pushing the key. The plectrum-string interaction begins at t_c, while t_r and t_f are the instants where the string and the key are released, respectively.

Then, during the plucking action ($\forall t \in [t_c; t_r]$), the plectrum velocity presents a second shape. Eventually, from the instant the string is released until t_f, the plectrum velocity presents the same pattern than during the first phase. After this last instant, the finger releases the key toward its rest position.

3.3 Initial condition of the string vibration

Although the free oscillations of the string do obviously not convey all the informations relative to the radiated sound, they reveal variations between plucking actions. Indeed, assuming the string flexible, stretched to a tension T, of uniform linear density ρ_l and fixed at its end, the transverse vibration of the string along the vertical direction can be described by [19, 20]

$$\begin{aligned} \vec{r}(z,t) &= \sum_{n=1}^{\infty} (A_n \cos(2\pi f_n t + \Psi_n) \\ &+ B_n \sin(2\pi f_n t + \Psi_n)) \, \vec{\Phi}_n e^{-\alpha_n t}, \end{aligned} \tag{1}$$

where $f_n = nc/2L$ are the eigenfrequencies, Φ_n are the modal deflections, α_n are the damping coefficients and $\Psi_n = \arctan \alpha_n / \omega_n$. The modal amplitudes A_n and B_n are written as

$$A_n = \frac{2D_{t_r} \sin(k_n z_0)}{k_n^2 z_0 (L - z_0)}, \tag{2}$$

$$B_n = \frac{2V_{t_r} \sin(k_n z_0)}{k_n^3 z_0 (L - z_0) c}, \tag{3}$$

with k_n the wavenumber, and z_0 the plucking point along the length of the string. $D_{t_r} = \sqrt{x_s(t_r)^2 + y_s(t_r)^2}$ and V_{t_r} are the initial position and velocity of the string regarding its rest position (see Fig. 2), respectively. Further, the magnitude of each polarization of the string is of great importance relatively to the radiated sound. It can be estimated by using the initial angle between the two components $x_s(z_0, t)$ and $y_s(z_0, t)$ of the transverse string motion during the first vibration instants. It is referred to as Γ_{t_r}, as shown in Fig. 2.

Tab. 1 presents the initial conditions of the string's vibrations for the two performed sequences. Let us remind

	Slow	Fast
$x_s(t_r)$ (mm)	0.27 ± 0.02	0.28 ± 0.02
$y_s(t_r)$ (mm)	0.71 ± 0.07	0.90 ± 0.01
V_{t_r} (m/s)	0.06 ± 0.02	0.15 ± 0.01
Γ_{t_r} (deg)	17 ± 2	20 ± 1

Table 1. Initial conditions of the string vibration. The mean is computed on eight samples. The reported uncertainty represents a 95% confidence interval.

that the jack velocity was substantially increased for the faster sequence. As expected, the *tempo* does not influence the string position in the horizontal direction: this value is completely determined by the initial position of the string relatively to the jack. However, results indicate that the more the jack velocity, the higher the string is released. In the present case, we measure an elevation of the string of about 27 % in the case of the faster *tempo* relatively to the slower one. This obviously implies a higher string's velocity magnitude as well as a more important force applied by the string on the plectrum's end. The latter directly conveys to a higher plectrum's deformation and therefore to a more important initial polarization angle Γ_{t_r}.

4. COMPARISON BETWEEN MEASUREMENTS AND MODELING

In this section, a comparison between a mechanical model of the plectrum-string interaction and experimental results is presented.

4.1 Plectrum-string interaction model

The model of the plectrum-string interaction presented in this section is based on that of Perng and colleagues [7, 8, 12]. In this model, based on the assumption that no friction occurs between the string and the plectrum, the string is seen as a punctual force applied perpendicularly to the plectrum. The latter is considered as a cantilever beam with small strain, made in an isotropic material, and without any twisting motion. Another assumption is the uniformity of the plectrum section. However, as shown in Fig. 4, the plectrum is beveled under its tip to slip on the string when the jack falls down. This thickness variation induces a modification of the moment of inertia at the tip of the plectrum. In order to take it into account in the model, the plectrum is considered as a beam composed of two segments: the first one from 0 to L_1 with a moment of inertia of I and the second one from L_1 to L with a moment of inertia of $I/8$. With these assumptions, the deflection angle of the plectrum is

$$
\begin{aligned}
l \in [0 - L_1], \Gamma(l) &= \frac{F}{EI}\left(Ll - \frac{l^2}{2}\right), \qquad (4)\\
l \in]L_1 - L], \Gamma(l) &= \frac{F}{EI}\left(\frac{7}{2}L_1^2 - 7LL_1 + 8Ll - 4l^2\right),
\end{aligned}
$$

where E is the Young modulus of the plectrum material and F the force applied by the string at a distance l to the plectrum (see Fig. 2). The second Newton's law is applied on the string's element in contact with the plectrum during the interaction. Its projection along x-axis and y-axis are written as

$$
\begin{aligned}
\Delta m \frac{\partial^2 x_s}{\partial t^2} &= -Kx_s + F_{px}(l), \qquad (5)\\
\Delta m \frac{\partial^2 y_s}{\partial t^2} &= -Ky_s + F_{py}(l). \qquad (6)
\end{aligned}
$$

With a geometrical approach, K is computed from the string's tension T, length L_s and the plucking ratio β using $K = T/(L_S\beta(1-\beta))$. Eqs. 5 and 6 indicate that the plectrum-string interaction depends on the string's position on the plectrum l. This position can be extracted from the jack displacement (x_j,y_j) by computing at each time step the following system of equations (with the small angle approximation):

$$
\begin{aligned}
x_s(t) - x_j(t) &= \int_0^l 1 - \frac{1}{2}\Gamma(x)^2 dx, \qquad (7)\\
y_s(t) - y_j(t) &= -\int_0^l \Gamma(x) - \frac{1}{6}\Gamma(x)^3 dx. \qquad (8)
\end{aligned}
$$

In this way, at each time, the string position (x_s,y_s), the plectrum deflection Γ and the plectrum force F_p may be known.

4.2 Results

In order to compare experimental data to theoretical ones, the model is computed with parameters given in Tab. 2.

In Figure 7, the plectrum deflection at the release instant is shown. Unfortunately, all the plectrum's shape cannot be extracted because, at this instant, the string hides the plectrum tip. Nevertheless, theoretical results are found to be consistent with the experimental data. In this figure, we also present the model based on a beam composed of

String length L_s	47.5cm
String tension T	85.7N
Plectrum Young Modulus E	1.5GPa
Plectrum length L	3.5mm
Beginning of the bevel $L1$	2.9mm
Moment of inertia I	0.011mm^4

Table 2. Parameters of the plectrum and of the string.

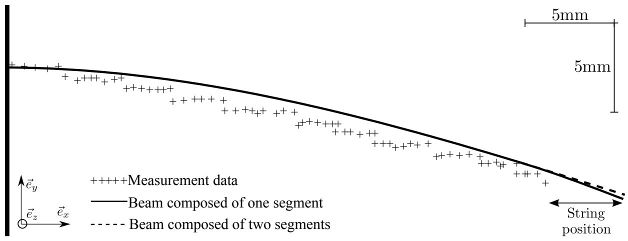

Figure 7. Experimental and theoretical plectrum deflection at the release instant. Referred axes are the same as in Fig. 2.

one segment of moment of inertia I with the one with two segments (as explained in Sec. 4.1). The proposed refinement induces a very slight modification of the plectrum shape prediction. However, the comparison of the plectrum deflection angle ($\Gamma(L) = \Gamma_{t_r}$) for the two configurations indicate that for an unique segment the angle is 13.4° whereas for two segments it is 20.9°. The second result is closer to the experimental value (see Fig. 1). The beam composed of two segments is thus valuable to model the harpsichord plectrum.

Based on the plectrum-string model, the initial conditions of the string vibration can be computed. At the release instant, we obtain $x_s(t_r) = 0.31$ mm and $y_s(t_r) = 0.80$ mm. These two values are very close to the experimental ones, especially for the fast case (see Tab. 1). Note that our results are obviously independent of the jack velocity because no friction force is taken into account in the model. Therefore, when the string slips fast on the plectrum, less friction forces occur and the model is found to be more appropriate than for the slow case.

5. CONCLUSION

This paper has presented an investigation of harpsichord plucking action. A well-controlled measurement setup conveying the jack and plectrum motion features as well as the produced sound has been carried out in a musical context.

As expected, results indicate that the plectrum-string interaction is longer when the playing *tempo* is slower. This is directly related to the jack velocity during the interaction. Indeed, a slower *tempo* leads to a lower jack velocity while it is in contact with the string. Besides, it has been shown that the plectrum velocity is constant during the plectrum-string interaction. Although these two parameters can be considered as control parameters, the instrumentalist cannot directly act on the initial conditions of the string vibration, which highly contribute to define the produced sound. However, a relationship has been pointed out between the playing *tempo* and the initial position, velocity and initial angle of the string's free oscillations. Indeed, we observed that the string is released higher relatively to its rest position for a higher playing velocity, which obviously imply a higher initial string's velocity. Moreover, the string applies therefore a more important force on the plectrum, conveying to a higher plectrum bending and then variation in the initial string's oscillations polarization. A deeper investigation on several plectrum's shape and material is required to conclude about the implications of these observations on the produced sound. Further, a plectrum-string interaction model has been implemented and confronted to our measurements. Simulations results indicate that the variation of the plectrum geometry along its length has to be taken into account in order to well-reproduce its deformations throughout the plucking action. This is of great importance to accurately predict the initial conditions of the string's vibration. The model is found particularly effective to give results very close to experimental ones, especially when the player plays rapidly.

Further work will be carried out to investigate harpsichord voicing. Several plectra of various shapes will be used for the measurements. A parametric study on the plectrum's geometrical parameters will be helpful to point out the relationship between the voicing process and the produced sound as well as the instrumentalist control. Further, refinements of the current plectrum-string interaction model will be proposed. For instance, the friction forces during the slipping phase will be modeled in order to improve the prediction of the initial conditions of the string free oscillations. Besides, as the plectrum-string interaction is obviously dependent on the musical context further comparative analyses will be performed on various playing techniques and music styles.

Acknowledgments

The authors acknowledge the Ministère de la culture et de la communication for its financial support to the project, as well as Jean-Claude Battault and Stéphane Vaiedelich for the fruitful discussions, Jean-Marc Fontaine for taking the pictures of the plectrum, Laurent Quartier for his technical help and Hugo Scurto for the preliminary tests.

6. REFERENCES

[1] M. Spencer, "Harpsichord physics," *The Galpin Society Journal*, vol. 34, pp. 2–20, 1981.

[2] N. Fletcher, "Analysis of the design and performance of harpsichords," *Acustica*, vol. 37, pp. 139–147, 1977.

[3] S. Le Moyne, S. Le Conte, F. Ollivier, J. Frelat, J.-C. Battault, and S. Vaiedelich, "Restoration of a 17th-century harpsichord to playable condition: A numerical and experimental study," *J. Acoust. Soc. Am.*, vol. 131, no. 1, pp. 888–896, 2012.

[4] S. Le Conte, S. Le Moyne, F. Ollivier, and S. Vaiedelich, "Using mechanical modelling and experimentation for the conservation of musical instruments," *Journal of Cultural Heritage*, vol. 13, no. 3, pp. 161–164, 2012.

[5] H. Penttinen, "On the dynamics of the harpsichord and its synthesis," *Proceedings of the 14th International Conference on Digital Audio Effects (DAFx-11)*, pp. 115–120, 2006.

[6] D. Griffel, "The dynamics of plucking," *Journal of Sound and Vibration*, vol. 175, no. 3, pp. 289–297, 1994.

[7] C.-Y. Perng, J. Smith, and T. Rossing, "Physical modeling of the harpsichord plectrum-string interaction," *Proceedings of the 13th International Conference on Digital Audio Effects (DAFx-10)*, 2010.

[8] ——, "Harpsichord sound synthesis using a physical plectrum model interfaced with the digital waveguide," *Proceedings of the 14th International Conference on Digital Audio Effects (DAFx-11)*, pp. 329–335, 2011.

[9] C. Sloane, "The effects of plectrum parameters on harpsichord sound," *Journal of Sound and Vibration*, vol. 125, no. 1, pp. 185–186, 1988.

[10] S. Carral and M. Paset, “The influence of plectrum thickness on the radiated sound of the guitar,” in *Proc. Acoustics ’08*, Paris, France, 2008.

[11] N. Giordano and J. Winans II, “Plucked strings and the harpsichord,” *Journal of Sound and Vibration*, vol. 224, no. 3, pp. 455–73, 1999.

[12] C.-Y. Perng, “Physical modeling of the harpsichord plectrum-string interaction,” Ph.D. dissertation, Stanford University, 2012.

[13] D. Chadefaux, J.-L. Le Carrou, B. Fabre, and L. Daudet, “Experimentally based description of harp plucking,” *J. Acoust. Soc. Am.*, vol. 131, no. 1, pp. 844–855, 2012.

[14] D. Chadefaux, J.-L. Le Carrou, and B. Fabre, “A model of harp plucking,” *J. Acoust. Soc. Am.*, vol. 133, no. 4, p. 24442455, 2013.

[15] S. El-Azim, “An efficient object tracking technique using block-matching algorithm,” in *Radio Science Nineteenth National Conference of the Proceedings of NRSC*, Alexandria, Egypt, 2002, pp. 427–433.

[16] C. Duxbury, J. Bello, M. Davies, and M. Sandler, “Complex domain onset detection for musical signals,” *6th International Conference on Digital Audio Effects (DAFx)*, vol. 1, pp. 90–94, 2003. [Online]. Available: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.57.9197&rep=rep1&type=pdf

[17] J. Bello, L. Daudet, S. Abdallah, C. Duxbury, M. Davies, and M. Sandler, “A tutorial on onset detection in music signals,” *IEEE Transactions on Speech and Audio Processing*, vol. 13, pp. 1035–1047, 2005.

[18] D. Chadefaux, “Interaction musicien/intrument : Le cas de la harpe de concert. (musician/instrument interaction: The case of the concert harp.),” Ph.D. dissertation, Université Pierre et Marie Curie, 2012.

[19] N. Fletcher and T. Rossing, *The Physics of Musical Instruments*, 2nd ed. New York, US: Springer, 1998.

[20] A. Chaigne and J. Kergomard, *Acoustique des instruments de musique*. BELIN, 2008.