Scaling of flute-like instruments : an analysis from the point of view of the hydrodynamic instability of the jet

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Abstract

The scaling of a family of five baroque recorders is studied considering two aspects: the compass of each instrument and the control parameters. The observations are interpreted in terms of the homogeneity of the timbre. The control parameters are measured on an experienced player performing a simple scale task on each of the instruments, and are described in the frame of the hydrodynamic jet behaviour.

On the family studied, the geometrical parameters appear to be adjusted so that the control parameters are similar on all the instruments. Low-pitched instruments present an enrichment of their spectra in high frequencies.

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1 Introduction

Consorts are known in the european classical music since the Medieval period as sets of instruments of like kind. The instruments of a consort offer different compass, allowing the development of polyphonic music covering a large tessitura (Galway [11]). Viols and recorders are probably the best known examples of consorts during the Medieval and Renaissance periods, but the idea of families of instruments has later been widely developed mostly for instruments showing a somehow restricted compass, like the flutes, the saxophones, the clarinets or the bowed string instruments.

Instruments of a given family generally share the same sound production mechanism, construction material and basic playing technique, while their dimensions are adapted to their individual compass. The design of a family should take into account the sounding homogeneity of the family but also some aspects of playability. Families are different according to their playability : while a recorder player will moslty practise on the alto recorder, s/he are expected to be able to play all instruments of the family from the bass to the sopranino. On the other hand, bowed string players are not expected to play other instruments of the bowed string family. A cellist, for example, will develop a technique to play the cello, and the maker does not have to make different instruments of the bowed string family that would be possible to play with a common technique. Coming back to the recorder player, it is most likely that s/he will prefer a family of instruments that can be played in the most similar ways.

This paper deals with families of flute-like instruments. The study was triggered by the fact that, from the point of view of the physics of the instrument, instruments at opposite ends of a same family should show some extreme adjustments of the excitation geometry and parameters. This is expected to develop our understanding of the making and playing compromises.

In the case of flute-like instruments, different families exist. The instruments can be classified according to the relative influence of the maker and the player in the resuting sound. In the case of organ pipes, the player decides the valve opening, that causes the pipe blowing, but the control parameters (*e.g.* the blowing pressure of the pipe) is determined by the maker. Conversely, shakuachi-like flutes are instruments in which the player not only controls the pressure and geometry of the excitation, but also the boundary conditions of the active end of the pipe, and has therefore a very wide control on the sound produced.

Study of an instrument family has already widely been discussed by Hutchins on the violin-strings family. The purpose there was to propose a new family made of eight members to cover the total range of written music. This study covers scaling rules between the different instruments and resulted in the making of a specific family : the violin octet [14].

Considering an organ rank as a flute family, scaling rules can be given. For instance, Bedos de Celles and Fletcher present rules for the different relevant dimensions of organ pipes [9, 1].

In this study a family of recorder is studied. In the recorder, the maker decides and builds the geometry, and the player focuses on the control of the blowing. One key difference with the violin family is that a single player is supposed to be able to play all the instruments of the family. The maker thus tries to maintain a similar technique along the family.

The goal of this study is to understand the way an instrument maker scales the different instruments of a family, keeping in mind homogenity of the playability as well as the resulting sound balance of the family. Control parameters in playing condition have been measured and are analysed, as well as some sounding properties.

Section 2 presents a quick overview of the current knowledge on the physics of flute-like instruments. The blowing indicator presented in this

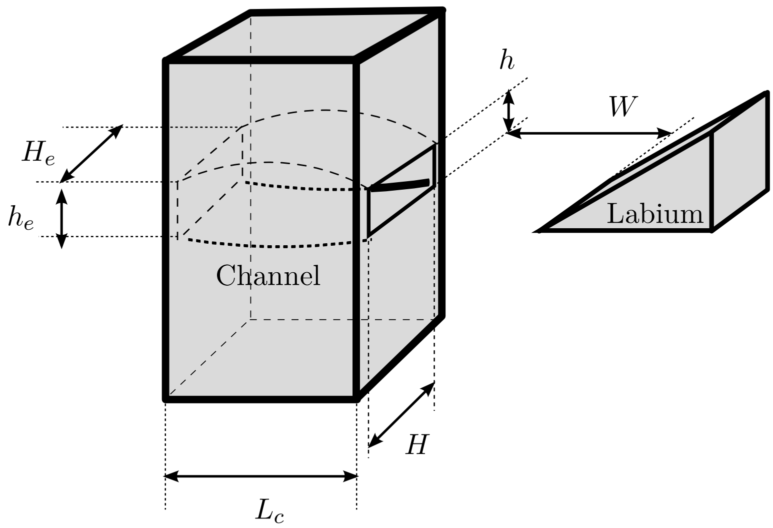

Figure 1: Schematics of the excitator of a flute-like instrument. The chamfers are not displayed, and the curvatures inside the channel are exagerated for readability

section will be used to analyse the player's control in section 4. Section 3 presents the geometry as well as the blowing characteristics associated with the five recorders of the family studied. Finally, the results will be discussed in section 5.

2 Framework

The excitation mechanism of a flute relies on the interaction between an unstable air jet and the acoustic field from the resonator. Perturbations are convected and amplified along the jet, and acoustic sources are created by the interaction between the jet and the labium [13]. The pipe acts as a resonator and accumulates energy at specific frequencies, resulting in a self sustained oscillation. Figure 1 shows a schematics of the excitator of a flute, defining the relevant dimensions we will use further : the height h of the channel, the distance W from the channel exit to the labium. The jet, with the center velocity u_j, is not presented on the figure 1.

In the literature, the instability of a laminar jet is described with the Rayleigh two dimentional instability theory [19, 8]. In his theory, the jet is inviscid and described by its stream function. A small perturbation with the pulsation ω is added to the jet stream function, resulting to a corrected flow. The perturbation stream function is of the form :

$$\left\{ \begin{array}{rcl} \psi(X,Y,T) & = & \Re\left(\phi(Y)e^{j(\alpha X-\omega T)}\right) \\ \alpha & = & \alpha_r + j\alpha_i \end{array} \right. \tag{1}$$

In the Rayleigh equation (1), $\phi(Y)$ and α are complex factors. The real part α_r of α is the wavenumber of the perturbation which propagates with the velocity $c_p = \frac{\omega}{\alpha_r}$. The imaginary part α_i stands for the spatial amplification of the perturbations along the path of the jet. Using mass and vorticity conservation expressions, and linearising, leads to a differential equation linking α, ω and ϕ to the unperturbed velocity profile of the jet. Thus, the amplification of the perturbations along the path of the jet depends on the frequency of the perturbation and the velocity profile of the jet.

At a first approximation, the perturbation convection velocity is proportionnal to the center velocity of the jet $c_p \approx 0.3u_j$. Coltman [4] stated that a phase optimum of the oscillation is reached when the time taken by the perturbations to travel through the mouth of the instrument $\frac{u_j}{W}$ is half the sound period$\frac{T}{2}$. A natural indicator of the blowing state of the instrument would then be the Strouhal number, which is the ratio of the frequency times the distance traveled between the channel and the labium and the jet velocity. In order to describe the blowing on an oscillation regime by a dimensionless number, we use the inverse of the Strouhal number, or dimensionless jet velocity, θ, as the jet velocity is directly linked to the mouth pressure control parameter (as discussed in section 3.2) :

$$\theta = Str^{-1} = \frac{u_j}{fW} \tag{2}$$

At the phase optimum defined by Coltman with the convection velocity of the perturbations, we have $\frac{0.3u_{j_{opt}}}{W} = 2f$ and thus $\theta_{opt} = \frac{u_{j_{opt}}}{fW} \approx 6$. The dimensionless velocity θ appears to be a blowing indicator, independently of the note played. In playing conditions, standard values are $7 < \theta < 17$, while θ may be as low as 3 for artificial blowing on the first oscillating regime. For high θ values, jet velocity is high relatively to the regime played. This eventually leads to a jump to the higher regime. On the other hand, for low values of θ, the oscillation can jump to the the lower regime or even stop.

Verge *et al.* [22] showed that the spectral content of the inner field of recorders strongly depends on the values of θ. In particular, for $8 < \theta < 10$, the amplitude of the second harmonic of the inner field presents a strengthening up to $20dB$, depending on the relative tuning of the passive resonances of the pipe, as discussed by Coltman [5]. Thus, θ is considered as a good descriptor of the state of the instrument.

Moreover, the spectral content of the inner field is tightly related to the radiated field's spectral content : in the experiments, the inner pressure p is measured at a distance Δx (taking into account the acoustic length corrections) of the exit of the pipe. As the waves in the resonator are stationnary, the amplitude of the oscillation measured is affected by the x-dependance of the pressure.

The complex amplitude of the acoustic field is then :

$$p_{in}(k) = \frac{p(\Delta x)}{\sin(k\Delta x)}, \tag{3}$$

where k is the wavenumber. In a low frequency frictionless approximation, the radiation of one end of the pipe can be described as a monopole, and is then related to the acoustic flow ϕ. The flow ϕ is simply derived from the inner pressure by Euler's equation, that leads to $\phi = -jS\frac{p_{in}}{\rho c}$, where S is the section of the opening. Levine & Schwinger [16], in Chaigne & Kergomard [2], derive a low-frequency approximation of the radiated field from a non-flanged pipe, that can be written, in the axis of the pipe :

$$P_{out}(r) = jk\rho_0 c\phi \left(1 + \frac{Z_R}{\rho_0 c_0}\right) \frac{e^{-jkr}}{4\pi r}, \tag{4}$$

where ρ_0 is the density of air at rest, c_0 is the acoustic celerity, and $Z_R = \rho_0 c_0 \left(\frac{1}{4}k^2\left(\frac{D}{2}\right)^2 + j\zeta k\frac{D}{2}\right)$ is the radiation impedance (Levine & Schwinger [16]).

P_{out} shows a highpass behaviour. Using equation (3) and under the assumption of low frequency ($k\Delta x << 1$), equation 4 can be written :

$$\tilde{P}_{out}(r) = S\frac{p(\Delta x)}{\Delta x}\frac{e^{-jkr}}{4\pi r}, \tag{5}$$

that is, at low frequencies the spectral content of the radiated pressure can be approximated by the spectral content of the pressure inside the pipe.

In this study a pressure sensor is flushed in the resonator at the position $x = D$ from the block, where D is the pipe diameter. The pipe diameter is small compared to the wavelengths of the acoustic waves λ. This means that $kD = \frac{2\pi D}{\lambda} << 2\pi$; with these parameters, comparing the approximation of the radiated field, $\tilde{P}_{out}(r)$, to the radiated field $P_{out}(r)$, the approximation proves to be valid within the range of $6dB$ up to $5000Hz$ for the bass recorder, and up to $14000Hz$ for the sopranino.

However, this model only takes into account the radiation at the blown end. A developement taking the radiation at the other end of a cylindrical pipe can be found in Chaigne & Kergomard [2]. Furthermore, the influence of the room should also be studied to predict the radiation of the instrument with accuracy. For these reasons, the internal field is considered in the study.

3 Description of the instruments

In the following a specific recorder family is studied. The recorders are baroque model *Aesthé*, made by the canadian recorder maker Jean-Luc Boudreau. The recorders are showed on figure 2. The frequencies of their lowest notes are $174Hz$ (F) for the bass, $262Hz$ (C) for the tenor, $349Hz$ (F)

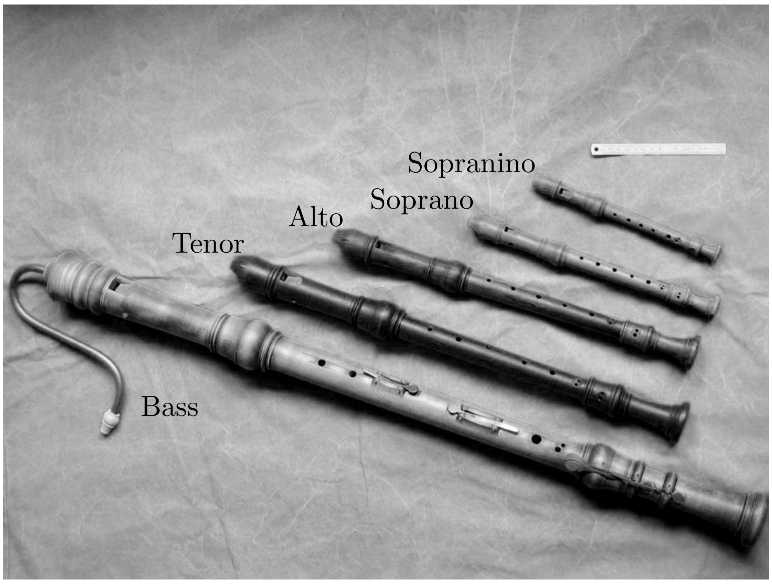

Figure 2: Photography of the five recorders studied. With the bass recorder, the player blows in a bocal, through the "S"-shaped pipe. In the experiments, the instrument is blown directly from the flow channel entrance.

for the alto, $523Hz$ (C) for the soprano and $698Hz$ (F) for the sopranino. A $15cm$ long ruler is also displayed as a scale reference.

Before studying the playing of a single player on the five recorders, we consider in this section their dimensions, as determined by the maker, and the resulting mouth pressure-flow characteristics.

3.1 Dimensions of the recorders

In the family studied, the sopranino recorder has a compass two octaves higher than the bass recorder. This means, from an acoustic point of view, that there is a four-fold decrease in the lengths of their resonators. It is obvious that the different dimensions of a recorder are not related to those of an other with a simple homothetic relation : if all the dimensions of the recorder were related by a factor of four between the biggest and the smallest ones, all the dimensions of the player would have to be related in such an order of magnitude to. The volume of air needed to blow the biggest recorder would then be sixtyfour times the one needed to blow the smallest.

The dimensions measured on the recorders are the lengths (L_c), the widths and the heights of the channel exits (H and h) and entrance (*resp.* h_e and H_e) , the distances between the channels and the labiums (W), the lengths of the chamfers and the diameters of the pipes (D). In the case of the bass recorder, the bocal is removed to get an acces to the entrance of the windway. The diameters of the pipes are measured in the cylindrical part

Recorder	Channel-labium distance (W) (mm)	Channel entrance width (H_e)(mm)	Channel exit width (H) (mm)	Channel length (L_c) (mm)	Pipe diameter (D) (mm)	Channel entrance height (h_e) (mm)	channel exit height (h) (mm)
Bass	7.5	21	19.24	61	32.1	0.98	0.85
Tenor	5.9	15.25	14.45	70.1	22.4	1.18	1
Alto	5.60	13.6	12.24	57	17.5	1.27	0.74
Soprano	4.45	10.22	9.48	44.7	13.2	1.08	0.67
Sopranino	4.15	7.48	7.50	34.8	10.6	1.25	0.8

Table 1: Measured lengths for the five recorders. Measurements of h and h_e are less accurate

of the bore, through a hole at the labium level. Please note that due to the size and the curved shape of the excitation region, measurements are less accurate for high pitched recorders, especially for the length of the chamfers. For the same reason, the measurements of the heights (h) of the channels exit are less accurate. However, one can note that they are always smaller than the heights of the entrances (h_e).

Table 1 shows the width H and length L_c of the channel and the distance W between the channel exit and the labium measured in the five recorders. These parameters show the greatest variation from an instrument to another. All the lengths measured decrease when the compass gets higher, excepted the channel length, that appears to be greater in the tenor recorder than in the bass recorder. The maker J.-L. Boudreau explains this is due to visual aesthetical reasons.

An efficient way to represent the variations of these lengths is to normalise them with the lengths measured on a reference recorder. The alto recorder is chosen as the reference, as its compass is situated in the middle of the compass of the whole family, and as it appears to be central in the making as well as in the practise of the players. The normalised measurements are shown on figure 3, with the inverse of the normalised fundamental frequencies of the lowest notes of each recorder : their ratios are the same as the acoustic lengths ratios. Excepted for the pipe diameter, the normalised lengths vary approximatively between $\frac{1}{2}$ and 1.5.

Figure 3 shows that the relations between the alto and the soprano recorders are nearly homothetic with a factor 0.8, while the dispersion between the different lengths is greater for the other instruments of the family. The further the compass of a recorder from the compass of the alto recorder, the greater the dispersion between the normalised lengths.

It is interesting to note that the normalised distance W is the length that

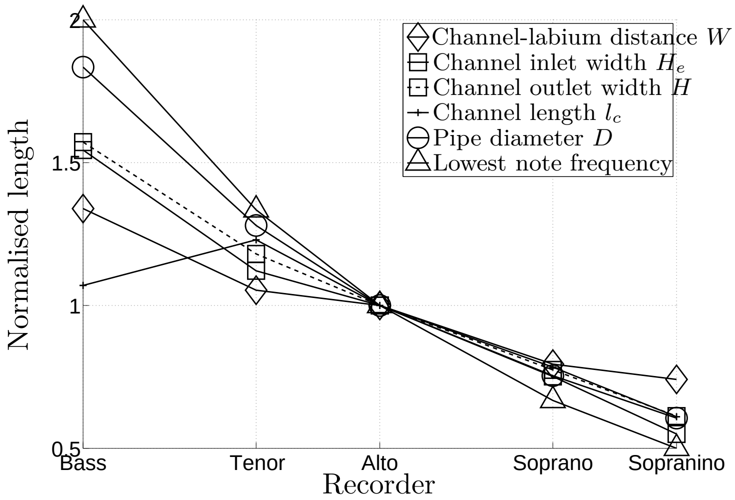

Figure 3: Normalised measurements of the recorders.

varies the less from one recorder to another. For low-compassed recorder, this length is reduced comparatively to the alto recorder. It is raised comparatively to the alto recorder for high-compassed ones.

The normalised pipe diameter varies between $\frac{1}{2}$ and almost 2. We know that the quality factor of the resonances of a pipe are tightly linked to its diameter. Thus, the diameter of the pipe has to follow the variations of the pipe length to ensure a constant quality factor. Each quality factor reaches a maximum at a given pipe radius, depending on the mode rank. For values smaller than this radius, viscothermal loss are predominant [18, 15, 2]. Over this radius, the loss is dominated by the open end radiations. A compromise has to be made in the pipe radius to ensure to have a sufficient quality factor for the first resonances of the pipe.

The mouth of the instrument presents a constriction and thus causes an acoustic length correction. The maker is then expected to adjust the mouth surface in relation to the section of the pipe. Figure 4 shows the ratios of the mouth surface and the pipe section for the five recorders, normalised with this ratio measured on the alto recorder.

Figure 4 shows that this ratio increases when the compass gets higher. This variation is quite slow, as the relative ratio increases of a factor two from the lower to the higher recorder. This means that the mouth surfaces are comparatively small in the low compassed instruments.

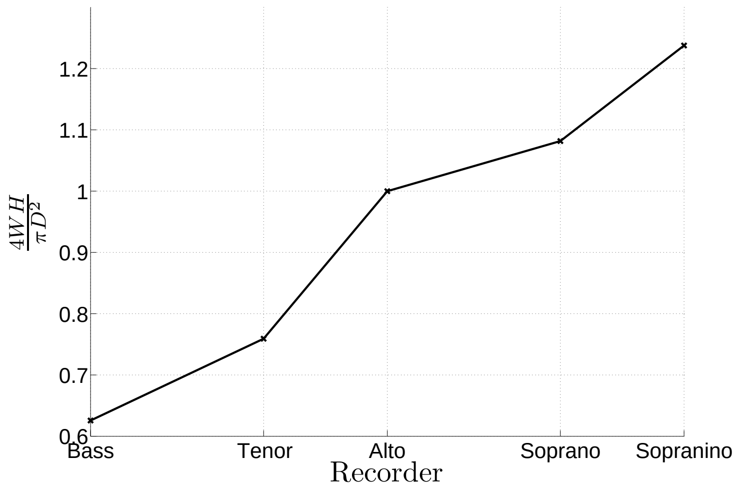

Figure 4: Ratio of the mouth surface and the pipe section for the five flutes of the family, normalised by the ratio for the alto recorder

3.2 Mouth pressure - flow characteristic

In its functionning, the recorder is excited by a flow, but the parameter controlled by the player is the pressure inside his mouth. Through measurements on flute players, Cossette *et al.* [6] showed that players use antagonistic muscles in order to control the air flow during the playing. This way, they can finely control their mouth pressure. In the recorder playing, the *resistance* of the channel helps to control the flow. The model usually admitted assumes that the jet velocity is given by the Bernoulli equation (6) :

$$u_j = \sqrt{\frac{2P_m}{\rho}}, \tag{6}$$

where P_m is the blowing pressure, measured in the mouth of the player.

This model assumes that there are no viscous losses in the channel. According to the players and the makers, the feeling of a *resistance* to the blowing is an important parameter, as it allows a finer control of the flow. The shape and the length of the channel determine the relation between the mouth pressure and the resulting air flow [21].

The relation between the mouth pressure and the flow entering the channel is measured for the five recorders of the family. Figure 5 shows the experimental setup used to measure the channel charateristics. The flow is controled with a flow regulator (Brooks 5851S) and the mouth pressure is measured with a manometer (Digitron 2020P). Figure 6 shows the mouth pressure-flow characteristics for the five recorder.

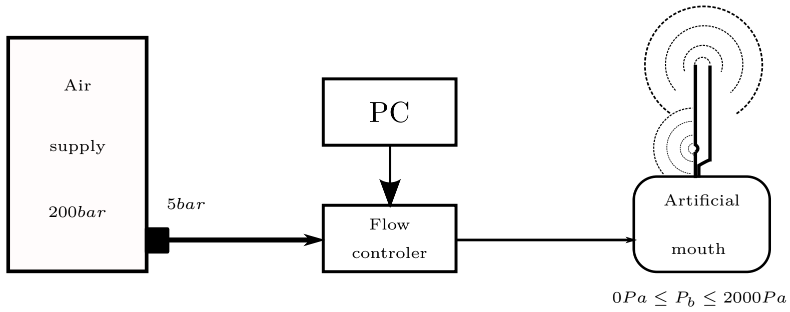

Figure 5: Experimental setup for the measurement of the mouth pressure - flow characteristic

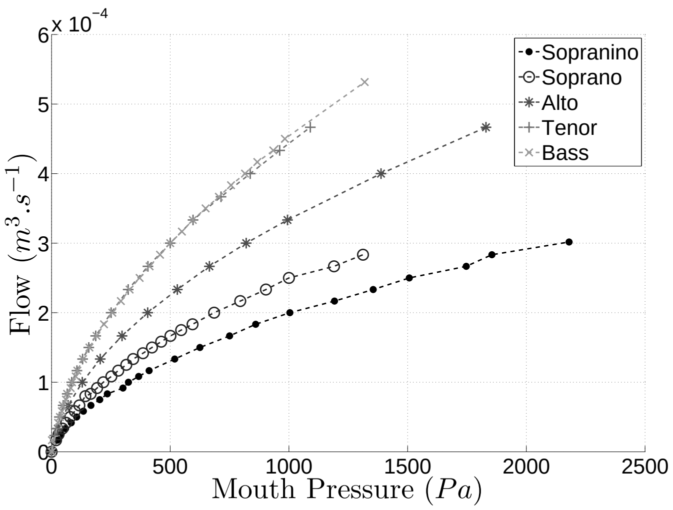

Figure 6: Air flow through the channel as a function of the mouth pressure

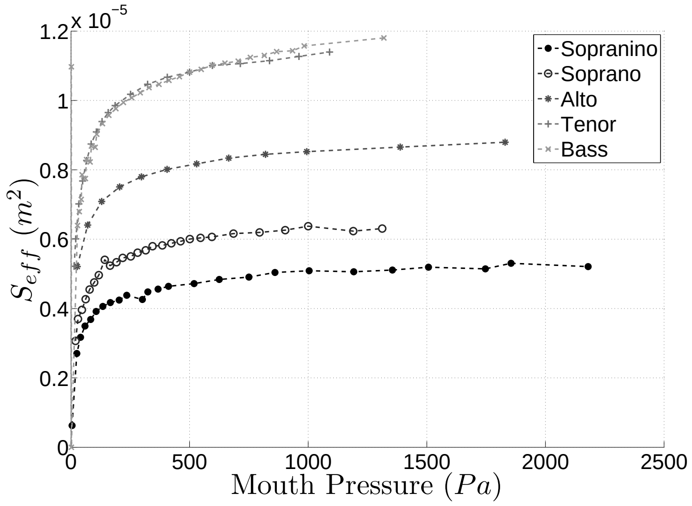

Figure 7: Pressure-flow (P - Q) characteristics normalised with the velocity computed from the Bernoulli relation, plotted as an effective cross section $S_{eff} = Q/\sqrt{\frac{2P_m}{\rho}}$

For a given pressure, the flows measured are quite different, from a recorder to another : the lower is the compass, the greater is the flow. This follows the measurement shown on figure 1, since the channels widths and heights are greater in the low compassed recorders. Considering that a low compassed recorder needs a greater flow to be blown than a high-compassed one, this result seems quite intuitive.

The figure 6 shows that the mouth pressure - flow characteristics of the Tenor and the Bass recorder are almost identical. This may be related to the blowing capacities of the players : a recorder requiring more air flow would not allow to play sustained notes.

In order to consider the characteristics in terms of *resistance*, flows are normalised with the theoretical flow given by the Bernoulli equation (6). Figure 7 shows the normalised pressure-flow characteristics. When the mouth pressure increases, the flow tends to behave with the Bernoulli's law, as observed by Martin [17]. In this representation, the curves tend asymptotically to an equivalent channel surface. Despite the great differences between the lengths of the different recorders, the rate of convergence with which the flow tends to behave like a Bernoulli flow is very similar in all the recorders.

4 Measurements on players

A specificity of the flutes families relies on the fact that a player is supposed to be able to play with any of the instruments.

A second step of the study is thus the measurement of the control parameters of a player in order to understand the adaptation from an instrument to the other.

4.1 Experimental set up

The instruments studied are tweaked in order to experiment with a player. A hole is made in the resonator close to the labium (one bore diameter from the block) so that the pressure sensor (B&K 4938) can be inserted to measure the inner pressure of the instrument. Along the channel of the instrument, another hole is put in order to have access to the mouth pressure of the player, with a calibrated differential piezo-resistive pressure sensor (Honeywell 176PC14HG1) placed inside the mouth of the player through a soft tube ($25cm$ long and $1mm$ internal diameter). The bandpass of the sensor stands between $0Hz$ and $2kHz$.

The player is asked to play several tasks, from scales to excerpts of musical pieces. Both the mouth pressure and the inner pressure are recorded, together with the radiated pressure. The radiated pressure is measured using mk 6 Schoeps microphones in omnidirectionnal mode, at a distance of around $60cm$ from the player. Recording the inner pressure limits the measurements to be disturbed by the acoustic of the room. Fundamental frequency detections are made using the YIN algorithm [7] on the inner acoustic pressure signal. The begining and end of each note is taken by hand.

4.2 Analysis of a scale

Playing scales is a typical exercise in learning an instrument, and is quite automatic after several years of practice. As no musical expression is involved, a scale is expected to provide standard control parameters.

The figure 8 presents the mouth pressure as a function of the note played for the five recorders, measured in the playing of a chromatic scale. The median and the distance between the first and third quartiles [20] of the mouth pressure are represented for each note. For a given recorder, the mouth pressure increases with the pitch. At the highest notes, a discontinuity of the mouth pressure is observed (this discontinuity may be easier to observe on the flow curves figure 9).

This discontinuity of the control pressure may be linked to a transition to turbulence. Table 2 shows the estimation of the Reynolds number of the jet $Re = \frac{u_j h_{eff}}{\nu}$, where ν is the cinematic viscosity of air, U_j is the velocity of the jet estimated from the mouth pressure with the Bernoulli relation (equation 6), and h_{eff} is the effective height of the windway exit, obtained by dividing the effective section S_{eff} by the channel width measured.

The Reynolds number is estimated for two notes : the note juste before

Recorder	Transition Reynolds	Highest note Reynolds
Bass	/	2047
Tenor	2371	2612
Alto	2239	2290
Soprano	2541	2028
Sopranino	2378	2930

Table 2: Estimation of the Reynolds number of the jet at the *transition note* and for the highest note of each recorder

the discontinuity, that we call *transition note*, and the highest pitched note of each instrument. Please note that the bass recorder does not present transition note. A transition to turbulence can be expected for $1000 \leq Re \leq 2000$.

The Reynolds numbers computed for the transition notes stand between $2200 \leq Re \leq 2700$. Again, one should note that using the Bernoulli relation to estimate the jet velocity leads to an overestimation of this velocity, and thus of the Reynolds number. It is interesting to note that the value of the Reynolds number computed with the highest pitched note of the bass recorder (which does not present discontinuity in the mouth pressure) is slightly lower than the Reynolds numbers computed for the transition notes of the other recorders. This may be a clue indicating that the jet in the bass recorder never reached the transition to turbulence in the experiments.

This discontinuity excepted, the logarithm of the mouth pressure seems to evolve linearly with the pitch, or, more generally, the logarithm of the frequency.

Considering the whole family instead of one recorder, it appears that the mouth pressure needed to play depends on the fingering rather than on the pitch. The overall pressure range stands between $300Pa$ and $3000Pa$. This is an important observation, as it means that despite the great differences of compass between the highest and the lowest recorders, the playing technique does not differ much in terms of blowing pressure. From the maker point of view, this can be seen as the way to help the player to adapt on the different recorders. However, the same pressure does not lead to the same flow in the different recorders (see figure 6).

Figure 9 shows the excitation flow of the different recorders for the same task. The flows are estimated by interpolation of the characteristics of figure 6 when the pressure range stands in the range of the characteristics measurement. Outside this range, the flow is estimated with the Bernoulli equation with the equivalent surface deduced from the data presented on figure 7.

In this representation, it becomes clear that on their whole compass, the recorders are excited on different registers. Especially for the highest notes,

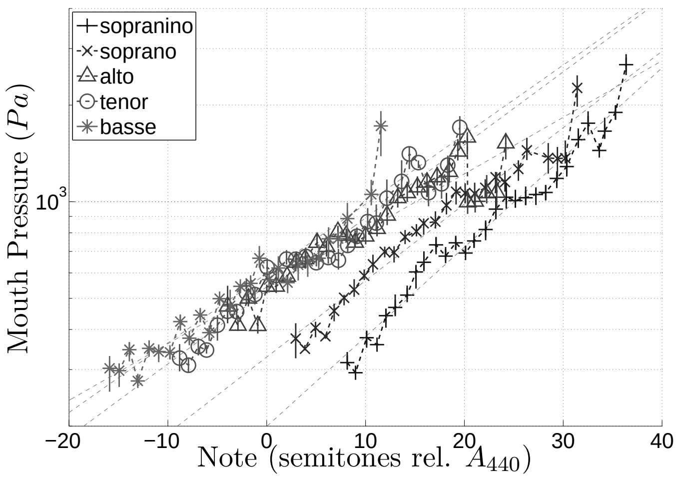

Figure 8: Mouth pressure versus note played for the five flutes in a chromatic scale playing

where a sudden decrease of the flow can be seen.

4.3 Recorder, pitch and spectral centroid

One key point in a family is to keep a sound unity among the different instruments. As with the lengths of the recorders, the spectra of the sound produced by two different recorders are not expected to be related with an homothetic relation. An homothetical relation between the spectra of the different recorders would result in a very dull sound for the low compassed ones, or a very piercing sound for the high compassed ones. In particular, the maker is expected to enhance the spectrum in the high frequency for the low pitched notes to preserve the audibility over the tessitura of the whole family.

We use the spectral centroid CGS (Grey & Gordon [12]) to describe the balance between high and low frequencies :

$$CGS = \frac{\sum_{f=0}^{\frac{Fs}{2}} f|A(f)|}{\sum |A(f)|}, \tag{7}$$

where $|A(f)|$ is the modulus of the spectrum,and Fs is the sampling frequency.

The spectral centroid increases with the frequency of the note played. A more remarkable result is that for a given note played with different recorders, the spectral centroid is of the same order of magnitude. As a consequence, the evolution of the spectral centroid over the tessitura of the

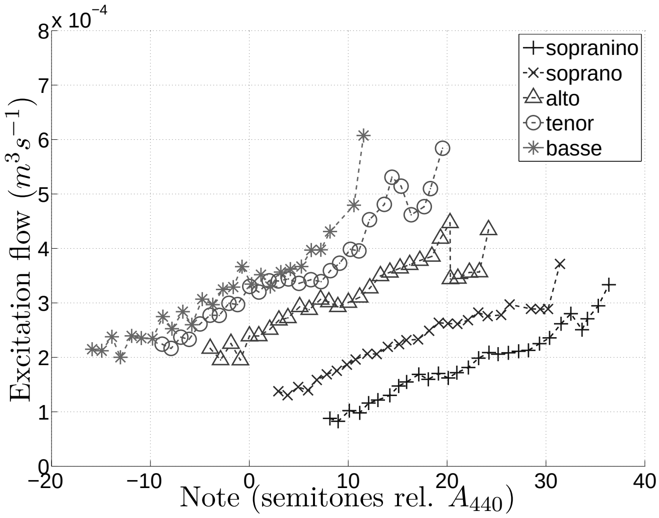

Figure 9: Excitation flow versus note played for the five flutes in a chromatic scale playing

whole family is continuous : there is no gap in the spectral centroid between recorders.

The gap between the lowest and the highest notes played in this study is 52 semitones, corresponding to more than 4 octaves, that is, a frequency ratio of more than 16 for the fundamental frequencies. On the other hand, the spectral centroid varies between approximatively $500Hz$ for the lowest note and approximatively $4000Hz$ for the highest note. Thus, the variations of the spectral centroid are less than a half of the pitch variations over the whole tessitura.

Figures 10 and 11 show the spectral centroid on the spectrograms of the inner pressure field of the bass and sopranino recorders. The frequency of the centroid is of the order of magnitude of the third harmonic for lowest notes of the bass recorder, while it follows the fundamental for the sopranino recorder.

It is noteworthy that in the frequency range of the spectral centroid presented in figures 10 and 11, the acoustic pressure radiated through the blown end can be approximated by the inner pressure, as discussed in section 2.

5 Discussion

In the making of a recorder family, some parameters have to be tuned in accordance with the physics of the instruments. Thus, the lengths and diameters of the resonators as the mouth surface depend merely on the

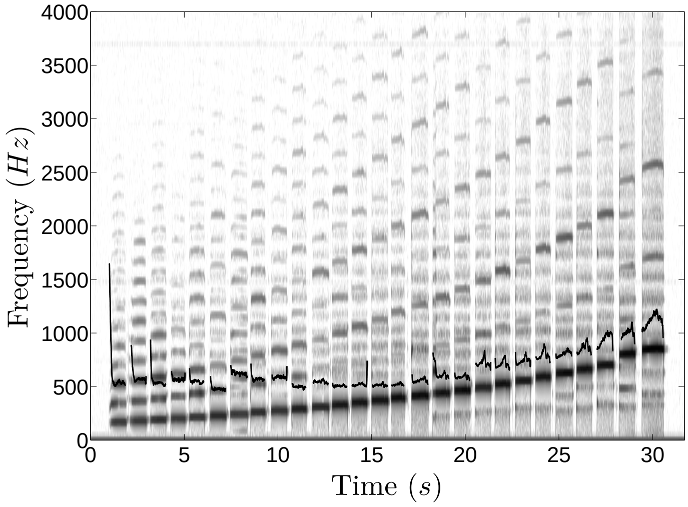

Figure 10: Representation of the spectral centroid (black line) on the spectrogram for the bass recorder

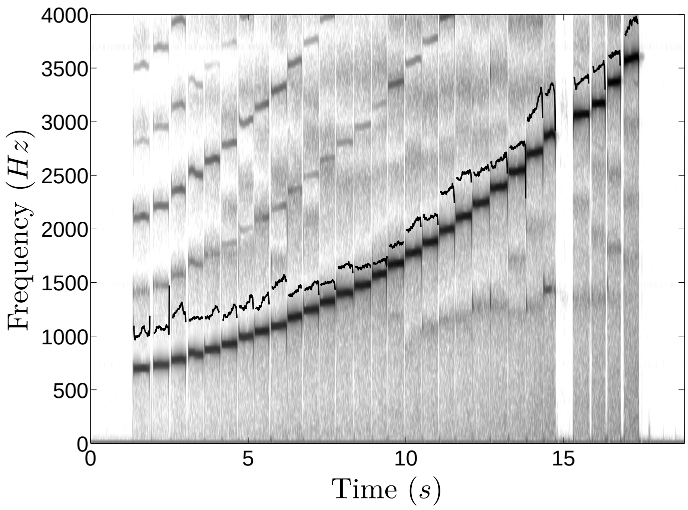

Figure 11: Representation of the spectral centroid (black line) on the spectrogram for the sopranino recorder

compass of each instrument. Other parameters can be tuned with more freedom and reflect the will of the maker, and some are related to the human physiology.

5.1 Dimensions of the instruments

As already noticed the lengths of the different recorders of the family are not homotheticaly related. With an homothetic relation between the lengths of the recorders, each recorder would need to be played by a player whose capacities are related with the compass. In a first place, the maker has thus to tune the parameters of the instruments so that they can be played by the same player.

Figure 8 shows on a scale task that the mouth pressure used to play depends on the fingering of the instrument rather than on the played note. But considering these measurements together with the mouth pressure - flow characteristics (figure 6) shows that the incoming flow is different from an instrument to an other with the same mouth pressure (figure 9).

The maker can tune the required mouth pressure and the incoming flow independentely by tweaking the channel geometry and the jet width. This provides the *resistance* needed to increase the mouth pressure while playing. This *resistance* can be illustrated considering the normalised characteristics (figure 7) : for the playing range observed, the flow inside the channel is closer to a Bernoulli flow with recorders presenting a higher compass than with recorders presenting a lower compass, while the flow is greater with low-compassed recorders.

On figure 12, the diameters of the recorders (table 1) are compared with the pipe diameters of the Prestant organ stop of the *basilique de la Madeleine* in Saint-Maximin, as measured by Cheron [3]. The pipe diameters of the organ are fitted with Fletcher's empirical law [9]. It is remarkable that in the middle of their tessitura, the recorder pipe diameters fit very well with the organ pipe diameters. The representation of the recorder diameters assumes that the bores are cylindrical and does not take into account tone holes. Thus, it should be considered as a representation of the order of magnitude.

On the same figure, the results of the computation of the pipe diameter maximising the quality factor of the three first resonances of an open-open cylindrical pipe are also displayed. The detail of the computation is presented in appendix A.

The principle of this computation is not new and is based on the same principle as the calculus of the pipe diameter variation law derived by Fletcher & Rossing [10], that leads to similar results. However, the value of the quality factor Q_n of the nth resonance of the pipe is here directly estimated, and leads to the variation law of the pipe diameter that appears to be a power of the frequency.

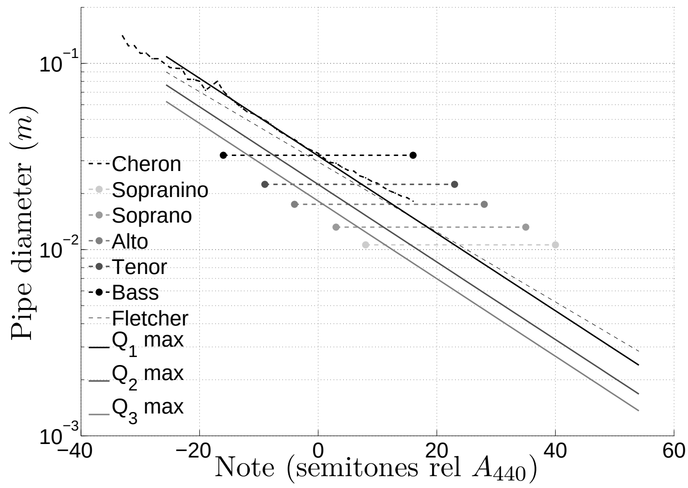

Figure 12: Comparison between the diameters of organ pipes with the diameters of the recorders versus the note played. The optimal diameters of cylindrical open-pipes, in terms of modes quality factors are displayed as well as Fletcher's empirical law for pipes diameters

In a simplified description of the oscillation in a recorder as a looped system (Chaigne & Kergomard [2]), the quality factor is an important parameter since it controls the slope of the phase shift around the resonances. This determines the frequency shift with the blowing blowing. The amplitude of the pipe response should as well be considered, especially around the oscillation threshold.

Moreover, considering a cylindrical pipe open at both ends may seem to be an oversimplification. Again, the aim of this estimation is to be as simple as possible, and adding the conicity, the constriction at the end of the pipe and the tone holes would be necessary for an accurate calculation for each fingering.

It is worth to note that the pipe diameters are of the same order of magnitude. Please note that the computation takes into account radiation and viscothermal losses in the pipe [16, 2], but neither the constriction in the active extremity of the pipe, nor the conicity of the recorder pipes.

Considering figure 12, it seems that pipe diameters are greater than the diameter maximising the quality factor of any pipe mode. For diameters greater than the diameter maximising the quality factor of a mode, losses are dominated by sound radiation. The recorders diameters plot do not take account of the conicty of the resonator either.

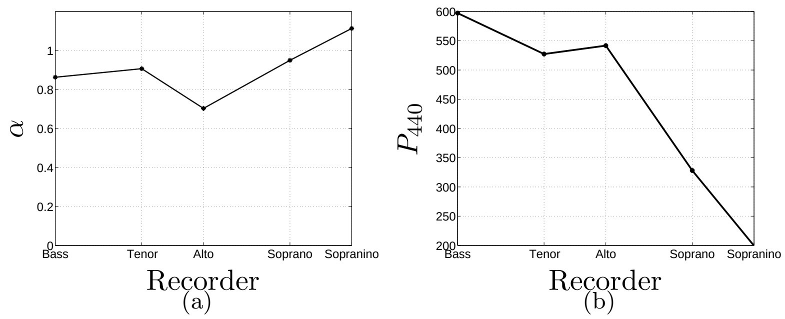

Figure 13: Parameters e^{α} and P_{440}, the reference blowing pressure at $440Hz$, of the fit of the mouth pressure

5.2 Mouth pressure

As noticed on figure 8, the natural logarithm[1] of the mouth pressure evolves linearly with the note played (relatively to A_{440}) for each flute. This can be written as :

$$\ln P_m = A\left[12\log_2\left(\frac{f}{440}\right)\right] + \ln P_{440}, \tag{8}$$

where A expresses the slope of the linear fit, and P_{440} the mouth pressure used to play an A_{440} within the fit. Equation 8 can be rewritten as :

$$P_m = P_{440}\left(\frac{f}{440}\right)^{\frac{12A}{\ln 2}} \tag{9}$$

This is of course a very crude approximation, as it fits with the same curve the different blowing pressure needed for the different registers of the instrument. The exponent $\alpha = \frac{12A}{\ln 2}$ expresses the slope of the linear fit, and P_{440} is the reference blowing pressure at $440Hz$. The values of these parameters are displayed respectively on figures 13a and 13b.

As in the figure 4, the alto recorder seems to mark a breaking between the recorders : its reference blowing pressure appears to be slightly higher, and the slope of its playing pressure lower than one could expect considering the reference pressures of the other recorders.

5.3 Sound amplitude

For the purpose of being played together, the relative sound intensities of the recorders have to be of the same order of magnitude. Figure 14 presents the acoustic pressure amplitude, in dB SPL, measured inside the instrument as a function of the note played. Please note that the inner field is composed

[1]Please note that the figure 8 is plotted on a $\log_{10}$ scale

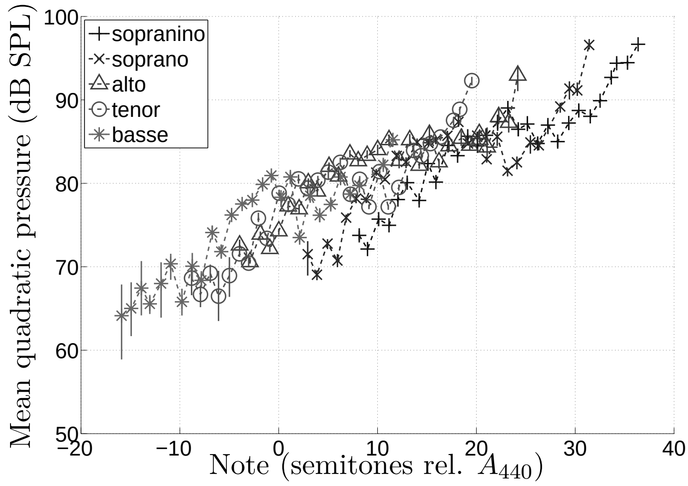

Figure 14: Inner mean quadratic acoustic pressure as a function of the note played, measured at a distance of one bore diameter from the block

by stationnary waves, and the amplitude is a function of the position of the microphone in the resonator. In all the recorders, the microphone is mounted, at a distance of one bore diameter from the block.

As shown in figure 14, the sound amplitude raises with the frequency within a range of $20dB$, and a slope of approximatively $10dB$ per octave. This might be linked to the fact that the source strength is, as a first approximation, proportionnal to the total jet flow. The mean quadratic pressure of the inner field presents also discontinuities at register changes with each recorder.

The radiated pressure has been recorded with a Schoeps mk 6 microphone couple in an acoustically untreated room. The microphones have not been calibrated and the intensity scale is relative to 1.

One key difference between figures 14 and 15 is the differences of sound pressure. While the slopes of the sound amplitudes versus the note played are of the same order of magnitude in the inner and radiated field, the differences between the recorders are reduced in the radiated field. One has to be careful with the interpretation of the radiated amplitude, as the room is acoustically untreated. Moreover, the instrument radiates through different holes, which leads to complicated interference patterns.

5.4 Dimensionless velocity

As already said, some of the making parameters of a recorder, as the pipe length, are determined by the physics of the instrument. On a second place, the making parameters can be used in order to tune the sounding of the

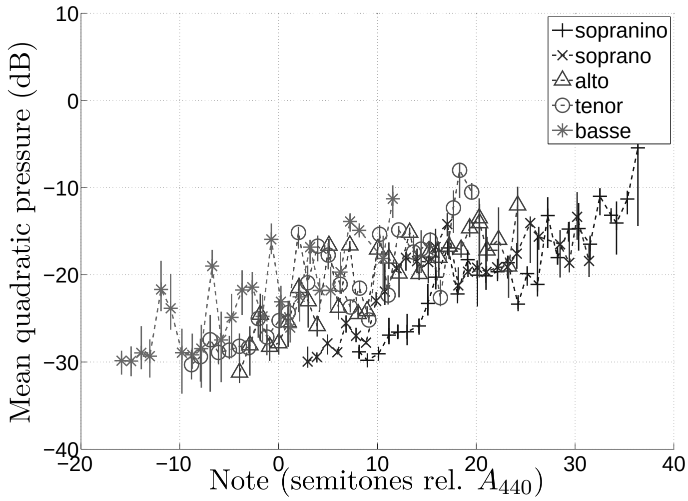

Figure 15: Mean quadratic pressure as a function of the note played for the radiated pressure, measured at a distance of $60cm$ from the recorder

instruments. Again, with homothetical relations between the excitation mechanisms of the recorder, the resulting spectra would be related with homothetic relations, leading to a dull sound for low compassed recorders.

Normalising the spectral centroid with the fundamental frequency measured shows an emphasis of the higher frequency for low-compassed recorders. Moreover, the normalised centroid tends to the unity for the highest notes (figure 16). As observed in figures 10 and 11, for the lowest notes, the normalised centroid tends to 3 and tends to 1 at highest pitchs. At a given pitch played with different recorders, the normalised spectral centroids are of the same order of magnitude.

It is noteworthy that the normalised centroid presents a change in slope around A_{440}. The centroid raises fastly when the pitch becomes lower, but decreases slowly when the pitch becomes higher than $440Hz$.

As discussed in section 2, the dimensionless velocity $\theta = \frac{U_j}{fW}$ is a good indicator of the blowing state of the instrument. In particular, spectral enrichment is observed when θ raises. It has been observed in section 3.1 that the distance between the channel exit and the labium, W, is comparatively short fo low compassed recorders than for high-compassed ones. Keeping the other parameters fixed, this leads θ to be relatively higher in low-compassed recorders than in high-compassed ones.

Figure 17 shows the dimensionless velocity of the jet as a function of the note played for the five recorders. The jet velocity is computed with the Bernoulli's relation, due to the lack of knowledge on the velocity profile of the jet and on the channel exit surface. Thus, the dimensionless velocity θ is

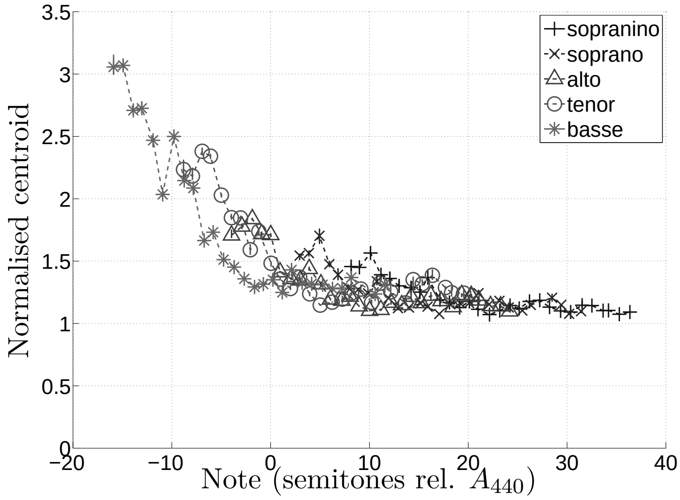

Figure 16: Normalised spectral centroid calculated on the inner sound field with the scale task

slightly overestimated. However, comparing the mouth pressures measured on figure 8 with the normalised characteristics on figure 7, leads to think that the approximation is good, except for the very low notes of each instrument.

Except for the bass recorder, the dimensionless velocities θ of the different recorders match for a given note. On the whole compass of the family, θ decreases linearly when the pitch of the note increases. Around A_{440}, values of θ are $8 \leq \theta \leq 12$. Below this pitch, the inner sound field presents an enrichment (figure 16), and the values of θ are consistant with the measurement by Verge *et al.* [22].

6 Conclusion

The study presented in this paper aims at understanding the characteristics of the different recorders that contribute to build a homogenous family through a greater than 4 octaves compass. The study is based on geometrical measurements on five handmade recorders designed to provide a homogenous family. The data is interpreted within the framework of the current knowledge on aeroacoustic sound production in flute-like instruments.

The main results indicate that the family is designed to provide an easy and homogenous control of the five instruments by using a common blowing pressure range, corresponding to a similar behaviour of the mouth pressure-flow characteristic. This may provide a homogenous feeling of *resistance* for the five recorders. The sounding homogenity of the family is controled both through a higher excitation flow in the low pitched instruments and an

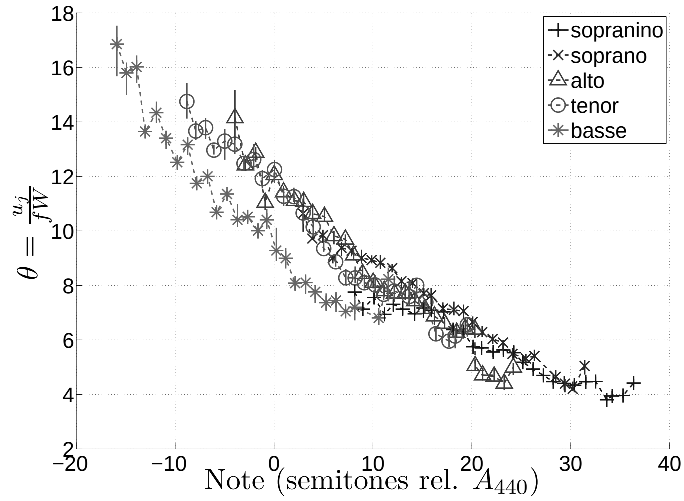

Figure 17: Dimensionless velocity θ as a function of the note played for the five recorders

increase of the spectral content for the low notes.

Our study is restricted to a specific recorder family and the result presented should be compared to others families in order to settle whether the ideas are general or specific to the family studied. Moreover, blowing and sounding parameters were studied for only one player. This also restricts the conclusions, eventhough the scale task studied here does not appear to be highly dependant on the players as far as mean blowing pressures are studied.

Acknowledgements

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A Estimation of the variations of the quality factor of a pipe with its radius

The passive resonances of an open-open pipe are studied here. At frequency close to the resonance frequencies of the pipe, a great part of the acoustic energy is kept in the pipe in the form of stationnary waves, while a small part of the energy is dissipated by two mechanisms : viscothermal losses near the walls of the pipe, and acoustic radiation at the extremities. The quality factor describes the ratio of energy kept by the pipe near a resonance frequency with the quantity of energy lost.

The quantity of energy loss affects the quality factor Q_n of the resonance modes of the pipe. Moreover, losses are dependent of the pipe radius : for narrow pipes, viscous losses are dominant. In the case of very wide pipes, the energy is lost through radiations.

We intend here to estimate the quality factor Q_n of the different resonance modes of an open pipe, as a function of the radius $a = \frac{D}{2}$ of the pipe. The following discussion is only valid for low levels, for which waves are governed by the linear acoustics laws. The solution is written using a perturbation method under the assumption of low frequencies $\frac{2\pi}{k} >> a$, and pipe radiuses large compared to the thickness of the boundary layers.
At low frequencies, plane waves travel in the pipe. For harmonic excitation, the acoustic pressure and velocity are of the form :

$$\left\{ \begin{array}{lcl} p(x,t) & = & (Ae^{-jkx} + Be^{jkx})e^{j\omega t} \\ v(x,t) & = & \frac{1}{\rho c}(Ae^{-jkx} - Be^{jkx})e^{j\omega t} \end{array} \right. \quad (10)$$

At low frequencies, the radiation efficiency is very poor, so that $|B| \approx |A|$ and the waves are stationnary. The acoustic impedance at the abscissa x is then written $Z(x) = -j\rho c \tan(Kx - \phi)$.

At $x = L$, the acoustic impedance is given by the radiation impedance (Levine et Schwinger [16]) ; then, writting the low frequency developement of the radiation impedance :

$$-j\rho c \tan(KL - \phi) = \rho c \left(\frac{1}{4}k^2a^2 + j0.6ka \right)$$

At low frequencies, the modulus of the radiation impedance is low compared to ρc. With a first order developement, we get :

$$-\phi = -KL + j\frac{1}{4}k^2a^2 - 0.6ka \quad (11)$$

At $x = 0$, the impedance of the pipe is a radiation impedance :

$$-j\tan(-\phi) = -\frac{1}{4}k^2a^2 - j0.6ka$$

Using the π-periodicity of the $\tan(x)$ function, and the low modulus of the radiation impedance we get :

$$\phi = j\frac{1}{4}k_n^2a^2 - 0.6k_na - n\pi \tag{12}$$

And combining equations 11 and 12, brings :

$$K_nL + 1.2k_na - j\frac{1}{2}k_n^2a^2 = n\pi \tag{13}$$

There is no viscous losses outside the pipe. The dispersion relation is simply $k_n = \frac{\omega_n}{c}$, where ω is the pulsation of the wave, and c the propagation velocity of waves. Inside the pipe, viscous losses and thermal transfer exist, and the dispersion relation of the waves is written $K_n = \frac{\omega_n}{c} + (1-j)\chi_n$,where $\chi_n = 3 \times 10^{-5}\frac{\sqrt{f_n}}{a} = \chi_n'\frac{\sqrt{f_n}}{a}$ (Chaigne and Kergomard [2]).

Equation 13 can then be rewritten :

$$\frac{\omega_n}{c}(L + 1.2a) + \chi_nL - j\left(\chi_nL + \frac{1}{2}\frac{\omega^2}{c^2}a^2\right) = n\pi \tag{14}$$

The quality factor Q_n is defined such as : $\omega_n = \Omega_n\left(1 + \frac{j}{2Q_n}\right)$. Which leads to :

$$\frac{\Omega_n}{c}(L + 1.2a) + \chi_nL + \frac{\Omega_n^2a^2}{2c^2Q_n} + j\left[\frac{\Omega_n}{2cQ_n}(L + 1.2a) - \chi_nL - \frac{\Omega^2a^2}{2c^2}\right] = n\pi \tag{15}$$

The real part of equation 15 can be approximated simply by $\Omega_n \approx \frac{n\pi c}{L+1.2a}$. Putting Ω_n in the imaginary part of equation 15, we get finally :

$$Q_n = \frac{\Omega_n}{2c}\frac{L + 1.2a}{\chi_nL + \frac{\Omega_n^2a^2}{2c^2}} \tag{16}$$

Figure 18 shows the variation of the quality factors of the five first resonances of a $64cm$ long pipe open at both ends, versus the pipe radius.

Searching a such as $\frac{\partial Q_n}{\partial a} = 0$ leads to :

$$-1.2\frac{\Omega_n^2}{2c^2}a^4 - 2\frac{\Omega_n^2L}{2c^2}a^3 + 2.4 \times \chi_n'\sqrt{\frac{\Omega_n}{2\pi}}La + \chi_n'L^2 = 0 \tag{17}$$

Figure 12 is drawn by solving equation 17 numerically for different pipe lengths.

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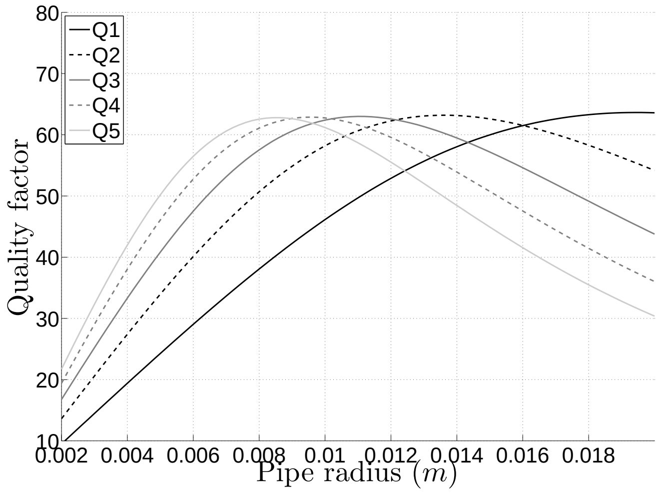

Figure 18: Quality factor Q_n of the five first modes of a $64cm$ long pipe open at its ends

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