Research internship : Asymptotic numerical approximations of tailored Green's functions for acoustic predictions - Application to sound scattering by a flutist's head

Keywords

Acoustics, Potential Flows, Fluid Mechanics, Flow Simulation, Green's Functions

Context and objectives

In playing conditions, a musician and its instrument are in close interaction. Thus, a complete analysis of musical instruments must include the effects of the musician's body on the instrument mechanical behavior. For instance, in some flute-like instruments, the pitch of the sound emitted by the instrument is dependant on the flutist's face position with respect to an open-end of the pipe, from where acoustic waves originate. An important feature of an instrument is its directivity, which is likely to be affected by the musician's body by scattering.

The propagation of acoustic waves in the absence of surfaces is easily described by the free-space Green's function G_0. When solid surfaces are present in the propagation medium, it is possible to find a Green's function with vanishing normal derivative $\partial G/\partial n = 0$. Such a function is said to be *tailored* to the geometry and allows fast estimations of the acoustic field in the presence of surfaces with arbitrary shape. Cases where the tailored Green's function is known analytically are however restricted to very elementary geometries. *The main objective of this internship is to determine asymptotic approximations of tailored Green's functions for shapes of arbitrary complexity in the long-wavelength (or low-wavenumber) limit.*

Indeed, acoustic waves that encounter a solid surface of size L have their energy redistributed in space by scattering, to an extent that is dependant on their wavenumber k. In many engineering and musical applications, acoustic sources are not only distant from the scattering object by less than its typical diameter ($r \sim \mathcal{O}(L)$), but they also emit at wavelengths greater than the object size ($kL \ll 1$). In such conditions, the scattering object is said to be *compact* and lying in the source *near-field*.

In this low-wavenumber limit, the theoretical framework of *compact Green's functions* introduced by Howe allows to write a first-order approximation of the tailored Green's G_t function as the sum of the free-space Green's function and a perturbation G' : $G_t \simeq G_0 + G'$, where G' can be determined by computing the velocity potential of the incompressible flow around the scattering object. This elegant mathematical framework allows, for instance, to derive an approximate tailored Green's function for the sphere as the potential flow around this geometry is known analytically. More complicated geometries can be considered using a numerical solver for the potential flow.

Main steps and methods

- Bibliographical study : Green's functions, matched asymptotic expansions, mathematical derivation of the low-wavenumber approximation
- Computation of potential flows on simple geometries using the Basilisk flow solver developed at ∂'Alembert and validation against analytical solutions
- Determination of the approximate Green's function frequency domain of validity with respect to analytical tailored Green's functions
- Application to a real case: scattering of the sound emission from a flute by a flutist's head

Practical information

Time : 5 to 6 months, start expected about March 2024.
Location : Institut Jean le Rond ∂'Alembert, Sorbonne Université, campus Pierre et Marie Curie, Paris, France.
Salary : Legal internship stipend ($\simeq$ 570 euros / month).
Profile : The candidate should be a Master's student in applied mathematics, acoustics, fluid mechanics or related fields. Knowledge of aeroacoustics would be much appreciated. They should have a taste for theoretical and numerical work, and for applied mathematics.
How to apply : The candidates must send CV (including recommendations if possible), grade records and a cover letter.
Supervisors : Antoine Hajczak (antoine.hajczak@sorbonne-universite.fr), Benoît Fabre (benoit.fabre@sorbonne-universite.fr)